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Low Reynolds number shear flow past a rotating circular cylinder. Part 2. Heat transfer

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Heat transfer from a cylinder placed symmetrically in a constant shear field is considered experimentally for low values of the shear Reynolds number, but for Péclet numbers Pe as large as 2000. With the cylinder held stationary, the experimentally obtained Nusselt number Nu is found to be in excellent agreement with the theoretically derived asymptotic expression, $Nu = 1{\cdot}641 Pe^{\frac{1}{3}}$ ($Pe \to \infty$), for Péclet numbers in excess of 100. In contrast, with the cylinder rotating at a speed corresponding to zero torque, the Nusselt number becomes effectively independent of the Péclet number for $Pe > 70$. This surprising behaviour, predicted theoretically by Frankel & Acrivos (1968), results from the presence of a region consisting entirely of closed streamlines which surrounds the freely rotating cylinder.

1. Introduction

The theoretical and experimental determination of heat and mass transfer rates from small particles suspended in a moving fluid has long occupied a foremost position in research dealing with transport processes. However, most of these studies have considered only the case in which the particles are assumed to remain stationary in a uniform free stream, with the result that little attention has been given to this problem when the particles are allowed to freely rotate and translate. The possibility that, under such conditions, the rate of heat and mass transfer would differ significantly from that of a non-rotating particle was recently pointed out by Pan & Acrivos (1968), who showed that in the limit of infinitely large Péclet numbers, Pe, conduction, rather than convection, remains the dominant mode of heat transfer for objects which are entirely surrounded by closed streamlines.

In order to verify this surprising conclusion, Frankel & Acrivos (1968) investigated theoretically the case of a heated circular cylinder freely suspended in a low Reynolds number uniform shear flow where, by virtue of its rotation, the cylinder is surrounded by closed streamlines, and showed that, in accordance with the general result of Pan & Acrivos, the Nusselt number became independent of the shear, and equal to 5·73, for asymptotically large Péclet numbers.

The profound influence exerted on the rate of heat transfer by the presence of closed streamlines can best be appreciated by recalling that, at low Reynolds

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numbers, the Nusselt number is proportional to $Pe^{\frac{1}{3}}$ in the limit of large Pe for flows past stationary surfaces (cf. Acrivos & Goddard 1965). The difference between the two cases is then easily understood by noting that, when all the streamlines emanate from upstream infinity, a thin thermal layer of thickness $O(Pe^{-\frac{1}{3}})$ is established adjacent to the heated surface through which the heat is transported, both by conduction and convection, to the main stream; in contrast, when the body is entirely surrounded by closed streamlines, heat can no longer be transferred to the bulk stream by convection but must instead diffuse slowly across these closed streamlines which, as shown by Pan & Acrivos (1968), also coincide with the isotherms.

Since the rather unusual theoretical conclusions arrived at by Pan & Acrivos, and by Frankel & Acrivos, have not, to date, been verified experimentally, it was decided to carry out a series of experiments using a heated cylinder, freely rotating in a shear field, in order to verify that the asymptotic Nusselt number is indeed independent of the shear for large values of Pe, and, furthermore, to try and establish the range of Péclet numbers for which this asymptotic theory remains at least approximately valid.

All the experiments to be reported were performed in the shear tank described in part 1, using as a test fluid the same polymer, Polybutene no. 8, which had served earlier for the flow measurements.

2. Heat transfer assembly

To obtain values of the Nusselt number experimentally, it is necessary that one be able to measure the heat flux and the temperature at the cylinder surface. In addition, the particular design chosen to accomplish this must also simulate the conditions for which the heat transfer theory was developed.

Specifically, the closed streamline patterns that are generated experimentally, must be identical to those obtained theoretically by solving the appropriate Stokes equation for flow past a cylinder freely suspended in a viscous, incompressible fluid, when the motion at large distances is that of a uniform shear. This was achieved in the shear-flow apparatus described previously in part 1.

Other assumptions which also must be taken into account when performing the heat transfer experiments are that the cylinder surface be isothermal, and that the heat be transferred radially to the main stream across the streamlines described earlier.

In order to meet these conditions, the heated portion of the cylinder must be suitably designed; however, this task is complicated by the fact that the heat transfer mechanism under investigation is basically one of conduction, and hence, other potential modes of heat transport, such as natural convection and axial conduction within the cylinder, must not be allowed to mask the effect that is being sought.

Free convection heat losses

Considering first the influence of free convection, we recall that, if the Prandtl number $Pr \gg 1$, as in this case, the non-linear terms which appear in the equations of motion may be neglected. As a result Nu_l, the corresponding Nusselt number based on the heated length, becomes a function of only one dimensionless group, the Rayleigh number Ra, where

$$Ra = Pr.Gr = \rho \frac{\nu c_p}{k} \frac{l^3 g\beta\Delta T}{\nu^2},$$

with Gr denoting the Grashof number. For temperature differences, ΔT, varying from 0·5 to 7 °C, typical values of the Rayleigh number to be encountered in the present investigation covered the range $200 < Ra < 3000$ where, as shown by Kutateladze (1963, p. 293), the experimental data available in the literature can be correlated by the empirical formula

$$\hat{Nu}_l = 1{\cdot}18 Ra^{\frac{1}{8}}. \tag{1}$$

Now since

$$\hat{Nu}_d = \hat{Nu}_l \frac{d}{l} = 1{\cdot}18 \frac{d}{l} Ra^{\frac{1}{8}} \tag{2}$$

(the caret denoting free convection), it is clear that if the heated length of the cylinder, l, is made too small, then the free convection contribution to Nu_d, the Nusselt number based on the cylinder diameter, will overshadow that due to radial transport. On the other hand, fabricating a cylinder with a long heated length in order to reduce $\hat{Nu}_d$ will result in the generation of velocities within the natural convection boundary layer comparable to those produced by virtue of the cylinder's rotation. This, of course, would severely disrupt the two-dimensional closed streamline pattern that was shown to exist in part 1, in the absence of heat transfer. In view of these two opposing effects, and the properties of the polymer,† a heated length of 1·0 cm was found to offer the best compromise.

Axial conduction heat losses

The problems associated with axial conduction losses from the heated portion were minimized by positioning guard heaters on either side of the heated test section. Since these were operated by a separate heating circuit, their power input could be varied independently from that of the test section until, ideally, the temperature gradient in the axial direction would be made to vanish at the junction between the guard and test regions.

Heat transfer cylinder

The next task was to formulate a design of the heat transfer cylinder which would transform these concepts into a working experimental assembly. First of all, since the heat had to be supplied electrically, the major difficulties to be resolved dealt with the problem of finding a suitable technique for transferring the heating current to and from the rotating cylinder as well as for obtaining the surface temperature. Due to the fact that the independent circuits for the guard and test sections required four electrical leads at the outset, it was decided to use the heated regions themselves as temperature detectors thereby eliminating the need to consider additional electrical connexions. This, of course, could be

† The properties of the polymer, Polybutene no. 8, were as follows (Robertson, 1969): $\nu = 4{\cdot}0$ Stokes at 20 °C, 2·0 Stokes at 30 °C; $\rho = 0{\cdot}852$ g/cm³; $k = 2{\cdot}97 \times 10^{-4}$ cal/cm sec °C; $\beta = 8{\cdot}67 \times 10^{-4}$ (°C)$^{-1}$; $c_p = 0{\cdot}47$ cal/g °C. Prandtl number, $Pr = 5390$ at 20 °C.

accomplished simply by coiling around the cylinder a suitable conductor having a large temperature coefficient of electrical resistivity, α, where

$$R = R_0(1+\alpha[T-T_0]),$$

R being the resistance of the conductor at temperature T. Recalling that resistance is defined as $R = \bar{\rho}L/A$, with $\bar{\rho}$ being the electrical resistivity, L and A the length and cross-sectional area of the conductor, it is clear that a material had to be selected which, in addition to having a large value of α, had to have a

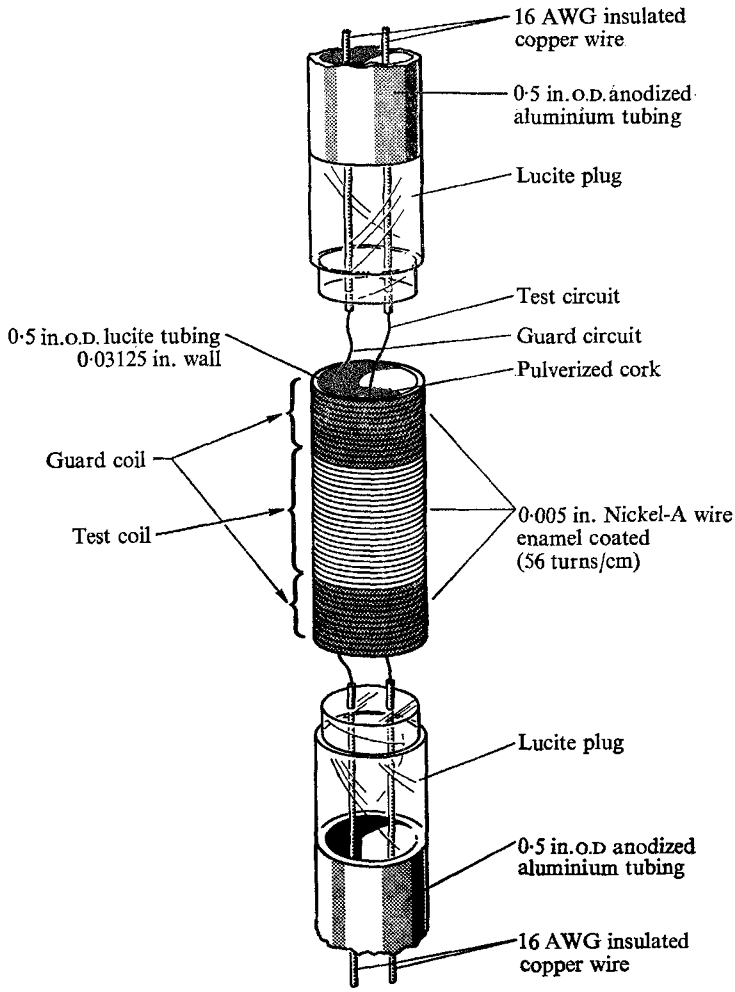

FIGURE 1. Heat transfer cylinder: test and guard coils.

high resistance in order to minimize the effect of the resistance of the leads. Extremely high purity Nickel-A wire, 0·005 in. in diameter with a 0·001 in. H-ML enamel coating, was chosen, having values of α and $\bar{\rho}$ equal to, respectively, $4{\cdot}3\times10^{-3}$/°C and 2·050 Ω/ft. at 20 °C.

As shown in figure 1, the wire was wound around a 2 cm section of $\frac{1}{2}$ in. O.D. lucite tubing having a wall thickness of $\frac{1}{32}$ in. The test coil covered a 1 cm section of the tube, and the guard coil the remaining $\frac{1}{2}$ cm on either side. The Nickel leads were then soldered to 16 AWG (0·0508 in. diameter) enamel-coated copper wires whose resistance was determined to be $2{\cdot}7 \times 10^{-5}\,\Omega$/ft.; hence it was obvious that changes in the resistance of the copper leads with temperature would be a negligible factor, i.e. less than $10^{-7}\,\Omega$/ft. °C. The soldered connexion was then coated with an insulating varnish to prevent the two circuits from contacting each other.

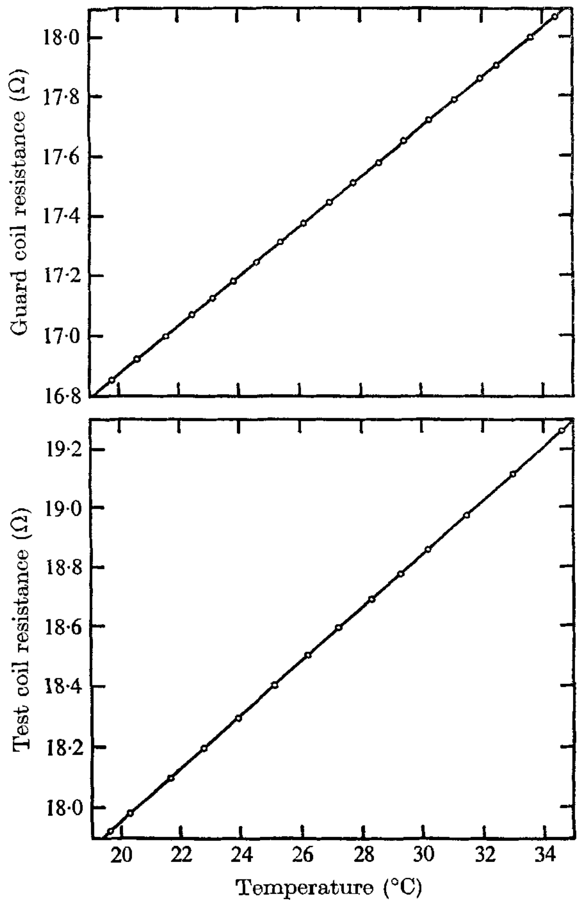

FIGURE 2. Resistance *vs.* temperature calibration curves: test and guard coils.

The interior of the tube was filled with loosely packed pulverized cork, thereby eliminating natural convection heat transfer to the interior. Following this, the tubing was glued at each end to lucite plugs in which the copper leads were permanently fastened to prevent any unnecessary strain at the very delicate junction between the nickel and copper wires. Two short sections of anodized aluminium tubing, $\frac{1}{2}$ in., O.D., $\frac{1}{10}$ in. wall thickness, were then fitted onto the ends of the lucite plugs as shown in figures 1 and 3.

Before completing the assembly the heat transfer cylinder was calibrated to obtain the resistance characteristics of the guard and the test coils. To this end, the cylinder was immersed in a constant temperature bath (± 0·02 °C), and the leads (either the test or guard circuit) were fastened to a Wheatstone bridge. The results of this calibration, shown in figure 2 for both the test and guard coils, indicate that the temperature coefficient of electrical resistivity was approximately $4\cdot9 \times 10^{-3}/°C$ at 20 °C. The importance in obtaining an extremely accurate calibration curve is clearly evident when one considers that a change in 1 °F corresponds to a variation of only 0·05 Ω in resistance.

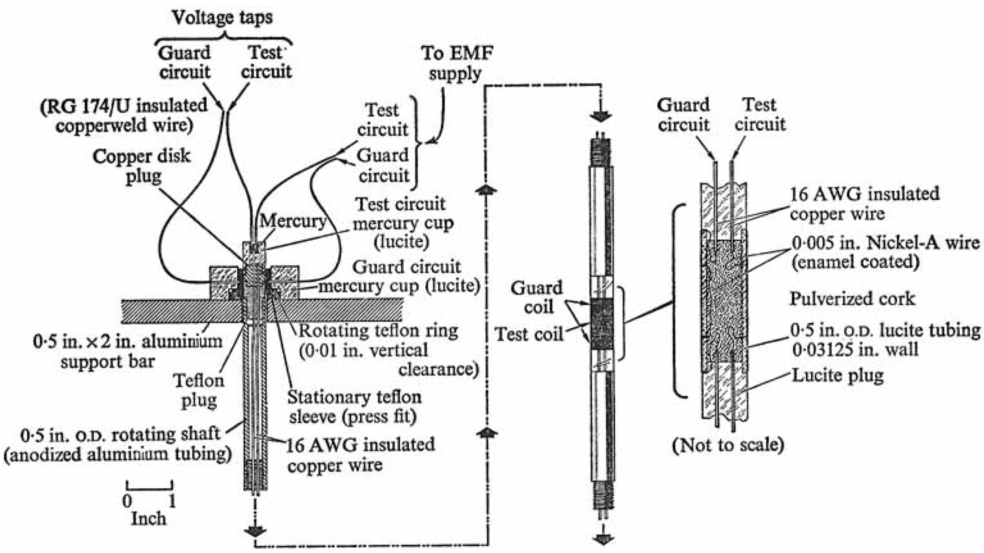

FIGURE 3. Heat transfer assembly: upper mercury cups and heat transfer cylinder.

Following this calibration, two more anodized sections of aluminium tubing were added to either end of the cylinder, and their lengths were selected so that the heated sections rested approximately midway between the air and water interfaces bounding the polymer in the test tank.

As shown in figures 3 and 4, each end of the cylinder was fitted with teflon plugs which served a dual purpose, as electrical insulators and as guides through which the copper leads passed. The guard leads at either end were soldered to $\frac{1}{2}$ in. diameter nickel plated copper disk-plugs, $\frac{1}{8}$ in. in thickness at the outer rim which, in turn, were fastened by screws to the teflon plugs. Since the guard leads were off-centre, the insulated test leads passed through the centre of the teflon plugs, the copper disk-plugs, and then through lucite plugs which were screwed directly onto the copper pieces.

The entire assembly was finally placed in the shear-flow apparatus. The upper portion of the cylinder was supported by an aluminium bar spanning the width of the tank, to which, a lucite mercury cup had been fastened. Also, provisions were made so that the height of the bar could be adjusted in order that the guard mercury well coincide with the rim of the copper plug, as indicated in figure 3, thereby allowing the current to be transferred from the rotating cylinder to the stationary external circuitry. To achieve a mercury seal and yet have a nearly frictionless, bearing surface, a very thin teflon ring was positioned beneath the guard mercury well, and then held in place (0·01 in. vertical clearance) by a larger press-fitted teflon ring.

In addition to providing a mercury seal, the smaller ring remained free to rotate with the cylinder, and also to move horizontally in order to absorb any effects, due to eccentricity caused by misalignment, without transferring the strain to the delicate heated section.

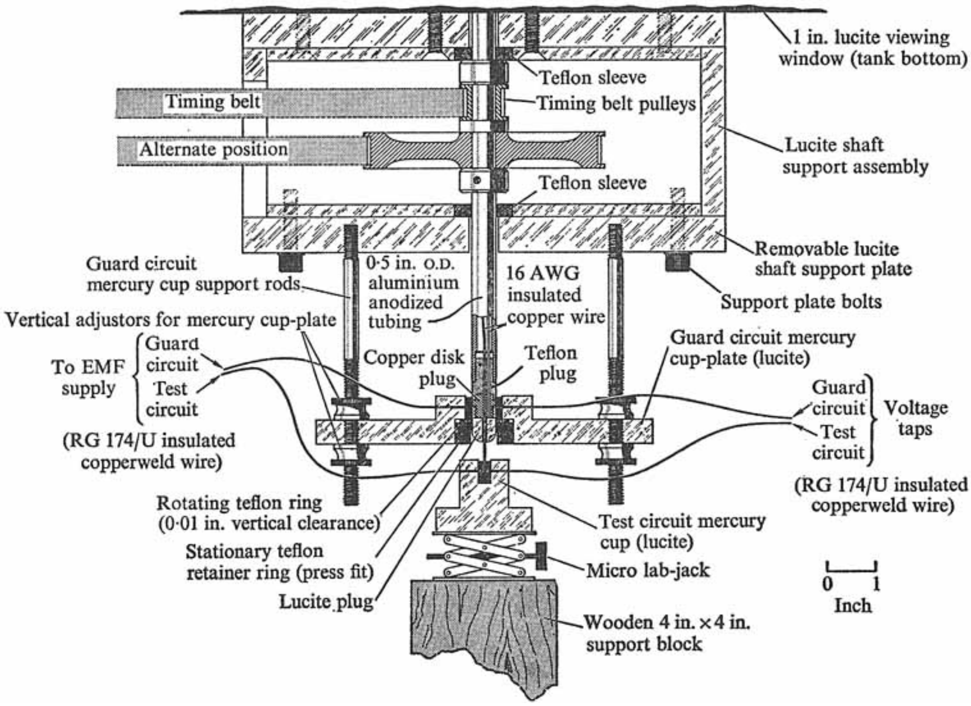

FIGURE 4. Heat transfer assembly: lower mercury cup.

To prevent deflexion of the cylinder due to the lateral strain induced by the belt and pulley system used to drive it, a lucite box was fastened to the bottom of the large viewing window of the shear-flow apparatus, as shown in figure 4. The pulleys and the locking collar for the rotating seal were fastened to the aluminium shaft inside the box structure, below which was suspended the lower guard mercury cup. As in the previous case, the mercury well could be adjusted in the vertical direction to effect alignment with the copper disk-plug conductor. The test circuit lead which passed through the copper disk and emerged from the lucite plug was immersed in another mercury cup whose height could be properly adjusted by using a micro-lab jack. This, of course, is in contrast to the upper test mercury cup, which was simply bored into the lucite plug as shown in figure 3.

External circuitry

We shall now deal with the external circuitry used to obtain the experimental data. Since the circuits for both the test and guard coils are identical, only a single schematic diagram, shown in figure 5, will be needed for reference.

The primary function of the circuit was to provide a means for measuring the Nusselt number Nu_d,

$$Nu_d = \frac{Qd}{Ak\Delta T} = \frac{i^2 R_c d}{Ak\Delta T\,(4{\cdot}186\,\mathrm{W/cal/sec})}, \tag{3}$$

where i is the current passing through the circuit (amperes), R_c is the resistance of the test coil (ohms), d is the cylinder diameter (cm), A is the area available for heat transfer from the test section (cm^2), k is the thermal conductivity of the polymer (cal/sec cm °C), and ΔT is the temperature difference between the bulk fluid and the cylinder surface (°C). In view of the circuitry, however, the Nusselt number may also be expressed as

$$Nu_d = \frac{E_s E_c}{\pi h k \Delta T\,(4{\cdot}186)}, \tag{4}$$

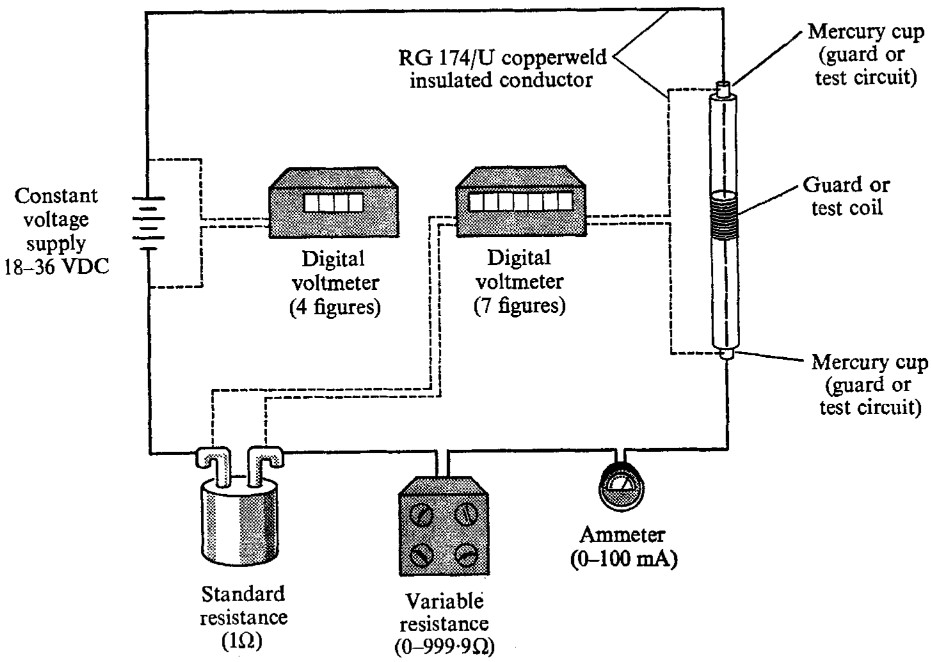

FIGURE 5. Heat transfer assembly: schematic circuit diagram.

where E_s and E_c denote the potential drops across the standard resistance and the test coil respectively, and h is the heated length of the test section. The technique used in gathering the data was the following: the Péclet number, defined as

$$Pe = Re.Pr = \frac{2\rho U_B a^2 c_p}{kH},$$

with a being the radius of the cylinder, was first determined knowing H, the distance between the belts, and the belt speed U_B (cf. part 1); the cylinder was then driven at an angular velocity corresponding to that of a freely suspended cylinder (see part 1), following which the test and guard coils were energized. After equilibrium had been reached, E_s and E_c were determined, and the average surface temperature of each coil computed by calculating the coil resistance E_c/E_s and then using either of the two calibration curves shown in figure 2. In most cases, H was kept fixed at 19·8 in. and Pe varied by changing the belt speed U_B.

It should be noted at this point that only average surface temperatures may be measured by using the resistance thermometry technique described above. Also, there appears to be no way of determining *a priori* the proper ratio of heat input to the guard and to the test coils in order to eliminate axial conduction from the latter. Obviously, using a low guard heat input would result in axial losses from the test coil, thereby lowering its surface temperature, whereas an excessive amount of guard heating would have the opposite effect. In either case, Nu_d is a monotonically increasing or decreasing function of the guard heat input for fixed values of Pe and the heat input to the test coil; hence, one can only resort to operating at several guard heat inputs in order to obtain a qualitative measure of these axial heat losses.

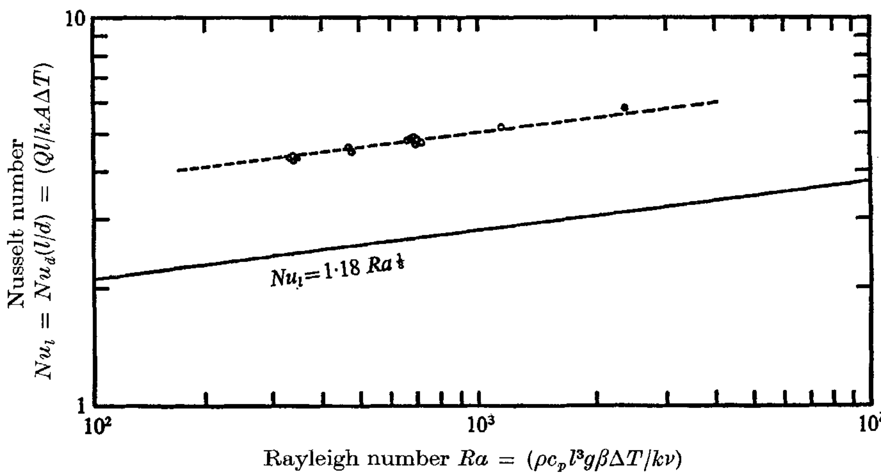

FIGURE 6. Natural convection heat transfer from a stationary vertical circular cylinder in the absence of guard heating.

3. Natural convection heat transfer

As a result of having realized that natural convection and axial conduction would, under most conditions, be a matter of some concern, it was decided initially to investigate the heat transfer characteristics of the experimental assembly for $Pe = 0$ in the hope of shedding some light on how these effects could be minimized or, failing that, how their influence on future studies could be ascertained.

The Nusselt number Nu_l based on the heated length was first determined as a function of Ra by operating the test coil in the absence of any guard heat over a range of ΔT from 1 to 10 °F, and the results are shown in figure 6. Clearly, for all practical purposes, the experimental data are in agreement with the empirical correlation (1), except for the proportionality constant which is too high.

However, since the Nusselt number decreased when the guard heater was used, these high values can be accounted for on the basis of axial conduction losses.

The only data shown on figure 6 are those obtained in the absence of guard heating. This is due to the fact the free convection correlation is valid only for isothermal surfaces, and not for surfaces along which the temperature varies, as was undoubtedly the case when both sections (guard and coil) were heated. Although not in quantitative agreement with (1), these results did indicate that the heat transfer assembly appeared to be operating as expected.

4. Heat transfer from a stationary cylinder

As stated above, due to the adverse effects of axial conduction heat losses, the free convection experiments were found to deviate significantly from the accepted correlation (1). Therefore, even though it appeared that the heat transfer assembly

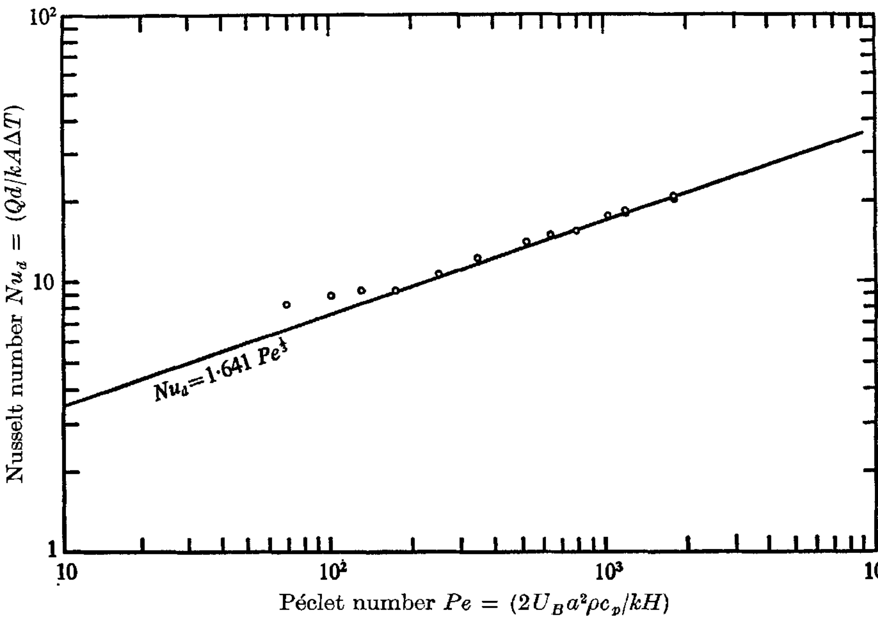

FIGURE 7. Forced convection heat transfer from a stationary vertical circular cylinder.

was performing as expected, it seemed desirable to carry out an experiment in which the contributions due to natural convection and axial conduction would both be much smaller than the primary mode of heat transport. Obviously, this required a forced convection experiment.

In order to accomplish such a task, it was decided to keep the cylinder stationary, thereby eliminating the closed streamline region which, as has been shown, is the primary reason why conduction dominates the heat transfer process even as $Pe \to \infty$. As shown in the appendix, the appropriate asymptotic solution of the energy equation leads, in this case, to the prediction that, if the surface is assumed isothermal,

$$Nu_d = 1{\cdot}641 Pe^{\frac{1}{3}} \quad \text{as} \quad Pe \to \infty. \tag{5}$$

The experiments were performed in the absence of any guard heating, on the assumption that forced convection would entirely overshadow both natural convection and the losses due to axial conduction. The results, displayed in figure 7, clearly indicate not only that these assumptions were correct, but also that the entire heat transfer assembly was performing quite satisfactorily.

Of course, as would be expected, the data tend to deviate from the theoretical curve as the Péclet number is decreased. This effect can be attributed to either the presence of free convection, or to the fact that, for $Pe < 10^2$, the asymptotic expression (5) can no longer be expected to apply, or to a combination of both.

On the basis of these preliminary studies, it seems quite reasonable to conclude, therefore, that the heat transfer cylinder together with the peripheral circuitry could serve as a very reliable system for obtaining the desired heat transfer data.

5. Heat transfer from a freely rotating circular cylinder

The following experimental procedure was adopted as a result of realizing that both axial conduction and natural convection would make significant contributions to the mechanism of heat transport when the cylinder was driven at the freely rotating speed ($\Omega = \frac{1}{2}$). For each value of the Péclet number, several values of ΔT were used, typically in the range of 1–3 °F, which, of course, gave an indication of the reproducibility of the data, and the effect of natural convection. Furthermore, a complete series of data was obtained for $2 < Pe < 350$, both in the absence of guard heat and in the presence of a guard heat to test heat ratio of 1·1. Clearly the former is the worst possible case with respect to axial conduction from the test area, whereas, in the latter, the guard heating used appears to be of approximately the right amount for the purpose of minimizing the axial heat losses.

It is evident from the results of these experiments, seen in figure 8, that the guard heating had the expected effect, namely, to cause a lowering of Nu_d. Here, the lines which appear in place of data points reflect the variation of Nu_d with ΔT at constant Pe, with the top and bottom of each line corresponding, respectively, to data obtained at the largest and lowest temperature difference that was used. Unfortunately, it was not possible to extend these experiments to $Pe > 350$, as had been done earlier with the stationary cylinder, for, when the gap H was decreased from 20 to 4 in. in order to obtain the desired high Péclet numbers without exceeding the maximum belt speed consistent with the proper operation of the shear-flow tank, it was found that the structure of the closed streamline region was significantly affected by the increased blockage. This was not the case when the cylinder was kept stationary since, there, the flow pattern appeared to be relatively independent of blockage within the range covered. Thus, it was felt that, under these conditions, the heat transfer data for the freely rotating case could not be properly compared with the theoretical predictions at these higher Péclet numbers.

The most important conclusion that can be drawn from figure 8, is the fact that, as $Pe \to \infty$, Nu_d does indeed become independent of the shear, in agreement with the theory of Frankel & Acrivos. Furthermore, it would appear this asymptotic theory is valid for Péclet numbers as low as 70, whereas, for small Pe, the value of Nu_d is in good agreement with that obtained from the free convection experiments for similar ΔT's. Unfortunately, owing to these free convection effects, it was not possible to test experimentally the theoretical expression for Nu_d at small Pe, originally derived by Frankel & Acrivos (1968), and recently extended to higher order (Robertson 1969), according to which

$$Nu_d = \frac{2}{1{\cdot}372 - \frac{1}{2}\ln Pe}\{1 + O(Pe^2 \ln Pe)\}.$$

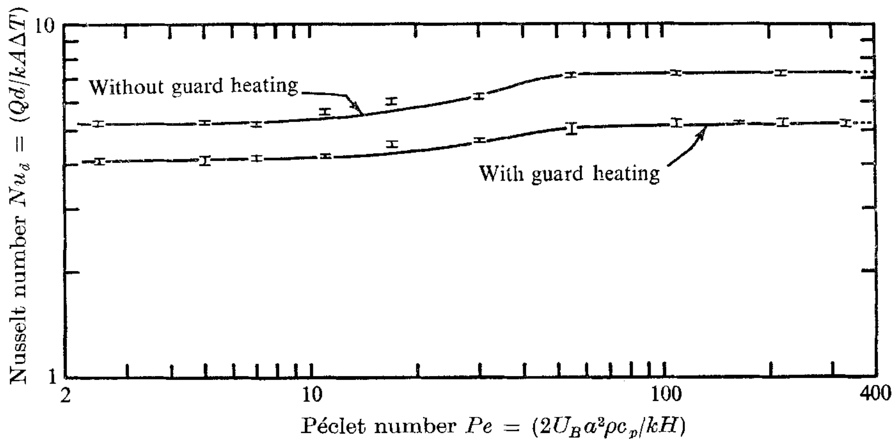

FIGURE 8. Heat transfer from a freely suspended circular cylinder in a uniform shear flow. Upper points: no guard heating; lower points: guard heat to test heat ratio of 1·1.

It is also worth noting that with a guard heat to test heat ratio of 1·1 the asymptotic Nusselt number was found to equal 5·3, in reasonable agreement with the theoretical value of 5·73 obtained by Frankel & Acrivos. In fact, a slight decrease in the amount of guard heating would probably have given the theoretical result, provided, of course, that natural convection effects are assumed to be of minor significance at large values of Pe.

At any rate, on the basis of the present experiments we can conclude that, as predicted theoretically, the heat transfer characteristics at high Péclet numbers of a small, freely rotating particle, are indeed fundamentally different from those that apply when the particle is held fixed, owing to the presence of a region of closed streamlines surrounding the heated object. Furthermore, it seems reasonable to expect that these conclusions will be valid also at higher Reynolds numbers for laminar flows having nonuniform velocity profiles in which the heated particles are freely suspended and thereby entirely surrounded by closed streamlines or stream surfaces.

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Appendix

The conservation equation governing combined diffusion and convection of heat is

$$\mathbf{u}.\nabla t = \frac{1}{Pe}\nabla^2 t, \tag{A 1a}$$

with boundary conditions

$$t = 1 \quad \text{on} \quad r = 1, \tag{A 1b}$$

$$t \to 0 \quad \text{on} \quad r \to \infty, \tag{A 1c}$$

where t is a normalized temperature, $\mathbf{u}$ the Stokes velocity field for a stationary circular cylinder in a uniform shear flow (see part 1, (4) with $\Omega = 0$, and figures 5 and 6), and Pe, the Péclet number. Since, as $Pe \to \infty$, the transfer of heat takes place across thin thermal boundary layers which originate at the stagnation points and then grow until they are eventually swept into the main flow, the Stokes velocity field can be approximated in the form of a Taylor series about the surface of the cylinder. Thus, for the purpose of obtaining an asymptotic solution to (A 1a)

$$u_\theta(y,\theta) = (2\cos 2\theta - 1)\,y + O(y^2),$$

$$u_r(y,\theta) = (2\sin 2\theta)\,y^2 + O(y^3),$$

where

$$y = r - 1.$$

Using the transformation of co-ordinates (cf. Acrivos & Goddard 1965),

$$Y = Pe^{\frac{1}{3}}y,$$

the proper balance of conduction and convection in the thermal layers is achieved, and hence, (A 1) becomes

$$2Y^2\sin 2\theta\frac{\partial t}{\partial Y} + (2\cos 2\theta - 1)\,Y\frac{\partial t}{\partial\theta} = \frac{\partial^2 t}{\partial Y^2} + O(Pe^{-\frac{1}{3}}), \tag{A 2a}$$

with

$$t(Y,\theta) = 1 \quad \text{at} \quad Y = 0, \quad t(Y,\theta)\to 0 \quad \text{as} \quad Y \to \infty. \tag{A 2b}$$

In terms of the similarity variables $t(x)$ and $g(\theta)$, where

$$x = Y/g(\theta),$$

it is easily seen from (A 2) that

$$\frac{d^2t}{dx^2} + 3x^2\frac{dt}{dx} = 0, \tag{A 3a}$$

$$t(x) = 1 \quad \text{at} \quad x = 0, \tag{A 3b}$$

$$t(x) \to 0 \quad \text{as} \quad x \to \infty, \tag{A 3c}$$

and

$$\frac{dg^3}{d\theta} - \frac{6\sin 2\theta}{2\cos 2\theta - 1}g^3 = \frac{9}{2\cos 2\theta - 1},$$

where the function $g(\theta)$ must remain finite at all points, except possibly in the thermal wake regions where the assumption of a thin thermal layer is no longer valid.

The appropriate solution to (A 3) is

$$t(x) = \frac{1}{\Gamma(\frac{4}{3})}\int_x^\infty e^{-\xi^3}\,d\xi.$$

Also, accounting for the development of the two different thermal layers (cf. figure 6, part 1),

$$g^3 = \tfrac{9}{2}(-\tfrac{1}{2}+\cos 2\theta)^{-\frac{3}{2}}\int_{-\frac{1}{6}\pi}^{\theta}(-\tfrac{1}{2}+\cos 2\xi)^{\frac{1}{2}}\,d\xi \quad (-\tfrac{1}{6}\pi \leqslant \theta < \tfrac{1}{6}\pi),$$

$$g^3 = \tfrac{9}{2}(\tfrac{1}{2}-\cos 2\theta)^{-\frac{3}{2}}\int_{\theta}^{\frac{11}{6}\pi}(\tfrac{1}{2}-\cos 2\xi)^{\frac{1}{2}}\,d\xi \quad (\tfrac{7}{6}\pi < \theta \leqslant \tfrac{11}{6}\pi).$$

Defining the Nusselt number based on diameter to be

$$Nu_d = -\frac{1}{\pi}\int_0^{2\pi}\frac{\partial t}{\partial r}\bigg|_{r=1}d\theta,$$

then

$$Nu_d = \frac{Pe^{\frac{1}{3}}}{\pi\Gamma(\frac{4}{3})}\int_0^{2\pi}\frac{d\theta}{g(\theta)},$$

which, in view of the above expressions for $g(\theta)$, becomes

$$Nu_d = \frac{6^{\frac{1}{3}}Pe^{\frac{1}{3}}}{\pi\Gamma(\frac{4}{3})}\left[\left\{\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}(\cos 2\theta-\tfrac{1}{2})^{\frac{1}{2}}\,d\theta\right\}^{\frac{2}{3}}+\left\{\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}(\tfrac{1}{2}-\cos 2\theta)^{\frac{1}{2}}\,d\theta\right\}^{\frac{2}{3}}\right].$$

Finally, upon numerical evaluation of the integrals, this reduces to

$$Nu_d = 1{\cdot}641 Pe^{\frac{1}{3}} \quad \text{as} \quad Pe \to \infty.$$

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